

TUM/T39-99-16

# Conformal string operators and skewed parton distributions<sup>1</sup>

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## Abstract

We discuss skewed parton distributions in the coordinate space. Solution of the corresponding LO evolution equation is constructed in terms of eigenfunctions of the evolution kernel and its relation to the conformal symmetry is explained.

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<sup>1</sup>Talk given at the XVth International Conference on Particles And Nuclei, June 10-16, 1999 Uppsala, Sweden

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Recently, there has been a renewed interest in QCD evolution of skewed parton distributions [1]. Skewed parton distributions play a crucial role in description of hard, exclusive QCD processes which are actively considered as tools for investigation of new aspects of non-perturbative QCD dynamics. However, it is clear that before non-perturbative information can be reliably extracted from experimental data, all perturbative aspects, such as QCD evolution, have to be understood. So far, the main effort has been devoted to studies of evolution of the skewed parton distributions (SPD) in the momentum representation [2].

SPD are defined through matrix elements of twist-2 string operators. Consider as an example a nonsinglet quark operator:

$$O(\alpha, \beta) = \bar{q}(\frac{\alpha+\beta}{2}z) \hat{z} \mathcal{P} \exp \left\{ -ig \int_{\frac{\alpha+\beta}{2}}^{\frac{\alpha-\beta}{2}} z_\mu A^\mu(tz) dt \right\} q(\frac{\alpha-\beta}{2}z), \quad (1)$$

Corresponding SPD can be introduced in the following way

$$\langle P' | O(\alpha, \beta) | P \rangle = \bar{N}(P') \hat{z} N(P) e^{-i\alpha \frac{r}{2} \cdot z} \int_{-1}^1 du F(u, \xi; \mu^2) e^{iu\beta(\bar{P} \cdot z)} + \dots \quad (2)$$

where dots denote other Dirac structures.  $N(P)$  and  $\bar{N}(P')$  denote initial and final nucleon spinors, respectively. The average nucleon momentum is denoted by  $\bar{P} = (P + P')/2$ , and the momentum transfer is  $r = P - P'$ . The asymmetry parameter  $\xi$  is defined by  $r \cdot z = 2\xi \bar{P} \cdot z$ .

Scale dependence of the SPD is governed by a generalised evolution equation

$$\mu \frac{d}{d\mu} F(x, \xi; \mu^2) = \frac{\alpha_S}{4\pi} \int_{-1}^1 dy V(x, y, \xi) F(y, \xi; \mu^2) \quad (3)$$

At the LO, the evolution kernel  $V(x, y, \xi)$  has a set of eigenfunctions associated with local conformal operators:

$$\int_{-1}^1 C_j^{3/2}(x/\xi) V(x, y, \xi) dx = \gamma_j C_j^{3/2}(y/\xi) \quad (4)$$

As these eigenfunctions do not form a complete set outside the region  $|x/\xi| > 1$ , they can not be used for expansion of the SPD. This can also be understood by the hybrid properties of the SPD. Let us split  $F(x, \xi, \mu^2)$  in two pieces:

$$F(x, \xi, \mu^2) = F_{<}(x, \xi, \mu^2) + F_{>}(x, \xi, \mu^2) \quad (5)$$

with

$$F_{<}(x, \xi, \mu^2) = \theta(x < \xi) F(x, \xi, \mu^2), \quad F_{>}(x, \xi, \mu^2) = \theta(x > \xi) F(x, \xi, \mu^2) \quad (6)$$

$F_{<}$ ,  $F_{>}$  describe partons with  $x < \xi$  and  $x > \xi$ , respectively. The crucial point is that the evolution of  $F_{<}$  and  $F_{>}$  is qualitatively different [1,6]. Partons which at the initial scale belonged to the segment  $0 \leq u \leq \xi$  stay there in the course of the evolution. On the other hand, partons which belonged initially to the segment  $\xi < u \leq 1$  diffuse into the segment  $0 \leq u \leq \xi$  and never come back. Mathematically this means that the function  $F_{<}(x, \xi, Q^2)$  will be restricted to the initial region  $0 \leq u \leq \xi$  but  $F_{>}(x, \xi, Q^2)$  will expand

to whole interval  $0 \leq u \leq \xi$ . The former and latter cases resemble the ERBL and DGLAP evolution, respectively. As it follows, properties of SPD in the region  $x > \xi$  SPD are similar to forward parton distribution  $f(x)$ , while in the region  $x < \xi$  SPD looks like a distribution amplitude. Expansion in the ortogonal set eigenfunctions (4) is valid for the ERBL-region  $x < \xi$  only, and reflects a typical structure of evolution for such configurations of partons. As we see, in momentum space there are two different regions in  $x$  with different evolution properties.

The situation is different in coordinate space. Coordinate-space SPD is defined through a Fourier transformation:

$$\mathcal{F}(\beta, \xi; \mu^2) = \frac{1}{\pi} \int_{-1}^1 dx F(x, \xi; \mu^2) e^{ix\beta} \quad (7)$$

It is easy to see that, unlike in the momentum space, the coordinate-space SPD  $\mathcal{F}_{<,>}$  associated with functions (6) are defined in the same interval  $0 \leq \beta \leq \infty$ . So, one can hope that an ortogonal set of coordinate-space eigenfunctions exists and can be used as a basis for expansion of SPD.

Evolution equations in coordinate space have been discussed e.g., in [3], where the role of the classical conformal symmetry was emphasized. The authors of [3] were able to write the solution in form of complex integral over conformal spin  $j$ . Recently, in [4] the solution have been obtained in terms of coordinate-space eigenfuctions corresponding to integer  $j$ . Here we will obtain solution for the SPD in coordinate space using a formal trick and then explain its relation with conformal symmetry.

Recall that a function  $f(x)$  can be expanded in a Neumann series according to [5]

$$f(x) = \sum_{n=0}^{\infty} (2\nu + 2n) J_{\nu+n}(x) \int_0^{\infty} \frac{d\lambda}{\lambda} f(\lambda) J_{\nu+n}(\lambda). \quad (8)$$

In particular, one finds that  $e^{ix\beta}$  can be decomposed according to the Sonine's formula [5]:

$$e^{ix\beta} = \left( \frac{2}{\beta\xi} \right)^{\frac{3}{2}} \Gamma[3/2] \sum_{n=0}^{\infty} i^n (3/2 + n) C_n^{\frac{3}{2}}(x/\xi) J_{\frac{3}{2}+n}(\beta\xi) \quad (9)$$

Inserting this expansion in the definition of the coordinate-space skewed quark distribution (7) and interchanging summation and integration one obtains:

$$\mathcal{F}(\beta, \xi; \mu^2) = \frac{1}{\sqrt{\pi}} \left( \frac{2}{\beta\xi} \right)^{\frac{3}{2}} \sum_{n=0}^{\infty} i^n (3/2 + n) J_{\frac{3}{2}+n}(\beta\xi) \int_0^1 du F(\omega, \xi; \mu^2) C_n^{\frac{3}{2}}(\omega/\xi). \quad (10)$$

Now, note that a Gegenbauer moment is proportional to the matrix element of multiplicatively renormalizable local conformal operator and its scale dependence is therefore given by

$$\int_0^1 d\omega F(\omega, \xi; Q^2) C_n^{\frac{3}{2}}(\omega/\xi) = L_{1+n} \int_0^1 d\omega F(\omega, \xi; \mu^2) C_n^{\frac{3}{2}}(\omega/\xi), \quad L_k = \left( \frac{\alpha_S(\mu)}{\alpha_S(Q)} \right)^{-\frac{\gamma(k)}{b_0}}$$

As it follows, the scale dependence of the coordinate space distribution  $\mathcal{F}(\beta, \xi; \mu^2)$  is given simply by

$$\mathcal{F}(\beta, \xi; Q^2) = \frac{1}{\sqrt{\pi}} \left( \frac{2}{\beta\xi} \right)^{\frac{3}{2}} \sum_{n=0}^{\infty} i^n \left( \frac{3}{2} + n \right) L_{n+1} J_{\frac{3}{2}+n}(\beta\xi) \int_0^1 d\omega F(\omega, \xi; \mu^2) C_n^{\frac{3}{2}}(\omega/\xi). \quad (11)$$

Now we show that equations (11), can be naturally understood as expansions of coordinate-space skewed quark distributions in terms of matrix elements of non-local, multiplicatively renormalizable, conformal operators. Indeed, applying (8) one can rewrite (11) as a Neumann-type series:

$$\mathcal{F}(\beta, \xi; Q^2) = \beta^{-\frac{3}{2}} \sum_{n=0}^{\infty} (3 + 2n) J_{\frac{3}{2}+n}(\beta\xi) L_{n+1} \int_0^{\infty} d\lambda \sqrt{\lambda} \mathcal{F}(\lambda, \xi; \mu^2) J_{\frac{3}{2}+n}(\lambda\xi) \quad (12)$$

We are now in the position to make the relation to conformal symmetry explicit. Let us start from the obvious identity

$$O(\alpha, \beta) = \int_{-\infty}^{\infty} d\alpha' \int_0^{\infty} d\beta' \delta(\alpha - \alpha') \delta(\beta - \beta') O(\alpha', \beta'). \quad (13)$$

Applying (8) one finds a representation of a  $\delta$ -function in terms of a Neumann series

$$\beta \delta(\beta - \beta') = \sum_{n=1}^{\infty} (1 + 2n) J_{\frac{1}{2}+n}(\beta) J_{\frac{1}{2}+n}(\beta'). \quad (14)$$

Inserting this expansion into (13) one finds that the string operator  $O(\alpha, \beta)$  can be decomposed as

$$O(\alpha, \beta) = \beta^{-\frac{3}{2}} \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{-ik\alpha} \sum_{j=1}^{\infty} (1 + 2j) J_{\frac{1}{2}+j}(|k|\beta) S(1/2 + j, k; \mu^2) \quad (15)$$

in terms of conformal string operators  $S(\frac{1}{2} + j, k; \mu^2)$ . Such operators

$$S(1/2 + j, k; \mu^2) = \int_{-\infty}^{\infty} d\alpha e^{ik\alpha} \int_0^{\infty} d\beta \sqrt{\beta} J_{\frac{1}{2}+j}(|k|\beta) O(\alpha, \beta), \quad (16)$$

introduced first in [3], form a representation of a conformal group and are therefore multiplicatively renormalizable at a one-loop level [3], i.e.

$$S(1/2 + j, k; Q^2) = L_j S(1/2 + j, k; \mu^2). \quad (17)$$

At this point  $j$  can be easily identified with the conformal spin. Taking matrix elements of both sides of the above equations one immediately reproduces equations (11).

Note that in the coordinate-space representation the corresponding LO amplitude  $M(\xi; \mu^2)$  can be written in following way:

$$M(\xi; Q^2) \propto i\pi \int_0^{\infty} d\beta e^{-i\beta\xi} \mathcal{F}(\beta, \xi; Q^2) \quad (18)$$

We have checked, using various models of skewed quark distributions, that the numerical algorithm for evaluation of physical amplitudes, based on equation (18), gives accurate and stable results, see Figure 1 for an example, except for a case where the variable  $\xi$

becomes small. This is related to the observation that a non-zero  $\xi$  provides a natural cut-off for large  $\beta$  behavior of coordinate-space distributions, which significantly improves the convergence of the Fourier integral (18), as compared to the forward case.

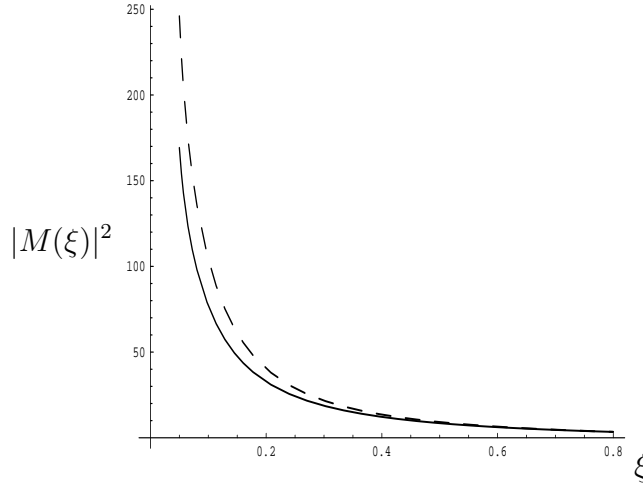


Figure 1. Typical results of evolution of  $|M(\xi)|^2$  as a function of  $\xi$ , starting from a  $\xi$ -independent initial conditions  $F(u, \xi; \mu_0^2) = 1.1641u^{-\frac{1}{2}}(1-u)^{3.5}$ . The solid line denotes  $|M(\xi)|^2$  at the initial scale  $\mu_0 = 1.777$  GeV, the dashed line represents  $|M(\xi)|^2$  evolved to  $\mu = 10$  GeV.

This work has been supported by AvHumboldt Stiftung.

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